The one-year non-life insurance risk

Esbjörn Ohlsson \(^a,^\ast\), Jan Lauzeningks \(^b\)

\(^a\) Länsförsäkringar Alliance, SE-106 50 Stockholm, Sweden
\(^b\) Gothaer Allgemeine Versicherung AG, Germany

**Abstract**

A major part of the literature on non-life insurance reserve risk has been devoted to the *ultimo* risk, the risk in the full run-off of the liabilities. This is in contrast to the short time horizon in internal risk models at insurance companies, and the one-year risk perspective taken in the Solvency II project of the European Community.

This paper aims at clarifying the one-year risk concept and describing simulation approaches, in particular for the one-year reserve risk. We also discuss the one-year premium risk and its relation to the premium reserve.

Finally, we initiate a discussion on the role of risk margins and discounting for the reserve and premium risk, with focus on the Cost-of-Capital method. We show that risk margins do not affect the reserve risk and show how reserve duration can be used for easy calculation of risk margins.© 2009 Elsevier B.V. All rights reserved.

1. Introduction

In most risk models, non-life insurance risk is divided into reserve risk and premium risk. Reserve risk concerns the liabilities for insurance policies covering historical years, sometimes referred to as the risk in the claims reserve (the provision for outstanding claims). Premium risk relates to future risks, some of which are already liabilities, covered by the premium reserve (the provision for unearned premium and unexpired risks); others relate to policies expected to be written during the risk period, covered by the corresponding expected premium income. (For technical reasons, catastrophe risk is often singled out as a third part of non-life insurance risk, but that lies outside the scope of this paper.)

The above-mentioned risks are also involved in the calculation of risk margins for the reserves; we will consider the Cost-of-Capital (CoC) approach, which is mandatory in the Solvency II Draft Framework Directive, EU Commission (2007). In the Discussion paper IASB (2007) on the forthcoming IFRS 4 phase II accounting standard, CoC is one of the listed possible approaches to determine risk margins.

In the Solvency II framework, the time horizon is one year, described by the EU Commission (2007) as follows: “all potential losses, including adverse revaluation of assets and liabilities over the next 12 months are to be assessed.”

In the actuarial literature, on the other hand, reserve risk has traditionally been discussed in terms of the risk that the estimated reserves will not be able to cover the claims payment during the full run-off of today’s liabilities, which may be a period of several decades; we call this the ultimo risk. If \( R_0 \) is the reserve estimate at the beginning of the year and \( C_\infty \) are the payments over the entire run-off period, this risk is measured by studying the probability distribution of \( R_0 - C_\infty \). This is the approach of the so-called stochastic claims reserving which has been developed in the actuarial literature over the last two decades, by Mack (1993), England and Verrall (2002) and many others.

Considering this background, it may not be surprising that it is noted in a study from the mutual insurers organization AISAM-ACME (2007), that “Only a few members were aware of the inconsistency between their assessment on the ultimate costs and the Solvency II framework which uses a one-year horizon”. Furthermore, the study shows that for long-tailed business, the ultimo (or as it is called there: full run-off) approach gives risk estimates that are 2 to 3 times higher than those for the one-year result. We conclude that it is both necessary and important to clarify the difference between the one-year and ultimo perspective.

In Section 6 of Dhaene et al. (2008), an approximate rule is given indicating that a one-year certainty level of 99.5% corresponds to a 40-year full run-off level of 81.8%. If we suppose that both risks are normally distributed, this can be seen to correspond to a volatility 2.8 times larger for the full run-off case, which is in line with the AISAM-ACME study. (On the other hand, Dhaene et al., at the end of Section 5, discuss an unclear point in the methodological description of the AISAM-ACME study that might explain some of the differences.)
On the methodological side, a few papers on the one-year reserve risk have recently appeared in the literature, see Wütrich et al. (2009) or Mez and Wütrich (2008). In the special case of a pure Chain Ladder estimate, they give analytic formulae for the mean squared error of prediction of the one-year result under an extension of the classic Mack (1993) model for the ultimo result. Wütrich and Bühlmann (in press) model the one-year risk when claims reserves are discounted.

Our first aim is to help in clarifying the methodological issues for the one-year approach to reserve risk, and in particular to describe a general simulation approach to the problem. A special case of this approach is the bootstrap methods in the context of Dynamic Financial Analysis which are implemented in some commercial software; cf. Björkwall et al. (in press).

In Section 3 we turn to the one-year perspective on premium risk, followed by a discussion in Section 4 on the role of the risk margin for premium and reserve risk. To the best of our knowledge, these issues have not been discussed in the literature before.

2. The one-year reserve risk

We will think of the one-year risk as being computed on January 1 of the new year. For \( t = 1, 2, 3, \ldots \), let \( C_t \) be the amount paid during year \( t \), with the new year denoted by \( t = 1 \). In order to separate between premium and reserve risk we will split \( C_t \) into \( C_t^1 \), paid for risk years previous to \( t \), and \( C_t^2 \), paid for the new risk year \( t \). Similarly, the closing claims reserve by the end of year \( t \), \( R_t \), is split into \( \tilde{R}_t \) for historical risk years and \( \tilde{R}_t \) for claims from the new year \( t \). Note that

\[ C_t = C_t^1 + C_t^2 \quad \text{and} \quad R_t = \tilde{R}_t + \tilde{R}_t. \]

We extend the notation to include \( R_0 \), the opening claims reserve for year \( t = 1 \). The reserve risk over a one-year time horizon is the risk in the claims development result (CDR), also called the (one-year) run-off result, which is

\[ \tilde{T} = R_0 - \tilde{C}_1 - \tilde{R}_1. \]  

(2.1)

Note that \( \tilde{T} \) is also the difference between the estimate of the ultimate cost for these risk years at times 0 and 1. The one-year reserve risk is captured by the probability distribution of \( \tilde{T} \). This is in contrast to the ultimo or full run-off risk, which was described above as the risk in \( R_0 - C_{\infty} \).

From the distribution of \( \tilde{T} \) we can compute any risk measure of interest. Here we will focus on the risk measure used in Solvency II: the Value-at-Risk at the level 99.5%, see EU Commission (2007, Article 100). Let VaR\((L)\) denote this risk measure for a loss \( L \), i.e. the 99.5% quantile of the loss distribution: if \( L \) is continuous then VaR\((L)\) is the solution to \( Pr\{(L \leq \text{VaR}(L)) \} = 99.5\% \). Here and below we assume that \( E(L) = 0 \). In the notation of Dowd (1998, p. 43) we say that we use the relative VaR.

Wütrich et al. (2009) condition on the observed part of the claims triangle. Along the same line, we will condition on any observations made up to and including year 0. We denote the collection of the corresponding random variables by \( \mathcal{D}_0 \); this might include data on paid, incurred or any other quantity that we could use for reserving in the particular application. This notation is extended to \( \mathcal{D}_t \), for \( t = 0, 1, 2, \ldots \). Let VaR\((L|\mathcal{D}_0)\) denote the VaR in the conditional distribution of \( L \) given \( \mathcal{D}_0 \). Then the one-year reserve risk (the risk in the CDR) is

\[ \text{VaR}(\tilde{T}|\mathcal{D}_0) = \text{VaR}(\tilde{C}_1 + \tilde{R}_1 - R_0|\mathcal{D}_0). \]  

(2.2)

Here we assume that \( E(\tilde{T}|\mathcal{D}_0) = 0 \), i.e. that \( R_0 = E(\tilde{C}_1 + \tilde{R}_1|\mathcal{D}_0) \) as it should be if we are using an unbiased estimate. With discounting and a risk margin this is no longer true, as will be discussed in Section 4.

The AISAM-ACME (2007) study distinguishes between the shock period, which is the one projection year when adverse events occur, and the effect period, which is the full length of the run-off of the liabilities. The direct effect in the shock period is captured by \( \tilde{C}_t \), while the variation in \( \tilde{R}_t \) relates to the effect period. Hence the one-year risk is, to some extent, affected also by the risk during the entire life-span of the liabilities, but not as much as the ultimo risk is.

Note. We tacitly assume that an expenses reserve is included in the claims reserve above. The question of how this should affect the total reserve risk is outside the scope of this paper. □

In risk models, the insurance portfolio is divided into more or less homogeneous segments, e.g. lines of business (LoB). To be able to calculate the reserve risk for a segment and combine it with other risks to form the total risk of the insurer, we need the probability distribution of the CDR \( \tilde{T} \) for this segment. Unless otherwise stated, the discussion below applies to the lowest level in the segmentation chosen in the risk model at hand.

2.1. Simulating the one-year reserve risk

In practical situations, unless normal distributions are used, simulation methods will often be the only possibility to assess the reserve risk. Here we will describe the steps in simulating the one-year reserve risk, in the case when we neither use discounting nor add a risk margin to the reserves. This simulation algorithm is by no means new, but, to the best of our knowledge, it has not been discussed in the literature before, except for a short description in a bootstrap context in a preprint of Björkwall et al. (in press).

2.1.1. Step 1: Best estimate of the opening reserve

Available is the actuary’s best estimate of the outstanding claims at the beginning of the year, the opening reserve \( R_0 \), which is treated as deterministic here since it is based on values in \( \mathcal{D}_0 \). In the Solvency II framework, reserves shall be the sum of a best estimate and a risk margin, where the best estimate is the expected present value of future cash-flows. We postpone a discussion of discounting and risk margins to Section 4, so in the present section the claims reserve is just the expected future cash-flows.

For a discussion on how to calculate the best estimate, see Groupe Consultatif (2008). We agree with this paper in that no detailed rules can be given for the choice of methods; here we will merely assume that the best estimate was computed by a documented algorithm \( A_1 \) that could be repeated in the simulations. For a long-tailed business such an algorithm might, e.g., be to use a development factor method (such as the Chain Ladder) on paid claims, with the factors smoothed and extended beyond the observation years by some regression model; then a Generalized Cape Cod (GCC) method may be used to stabilize the latest years, while the earliest years reserves might be adjusted somehow by the claims incurred. For a description of the GCC, see Struzzieri and Hussian (1998).

In our opinion, the quite popular Bornhuetter–Ferguson (BF) method does not fulfill the requirement of being algorithmic. The reason is that it uses some a priori loss ratios, whose calculation is outside the method. In practice, these loss ratios will probably be estimated from loss data and in order for BF to be algorithmic, this estimation should be made explicit. The GCC could be seen as a way of making BF algorithmic and is hence preferred here.

The use of a repeatable algorithm \( A_1 \) might be considered good practice, leaving as little as possible to the actuary’s subjective judgement on individual figures. If such judgement is still necessary, we have to find an approximate algorithm \( A_1 \), capturing the main features of the best estimate, for use in the simulation, Step 3.
Table 2.1

<table>
<thead>
<tr>
<th>Development triangle with simulated variables in bold-face; the other variables belong to $D_0$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin years</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>n-1</td>
</tr>
<tr>
<td>n</td>
</tr>
</tbody>
</table>

2.1.2. Step 2: Generating the new year

The next step is to simulate the events of the risk year, conditional on $D_0$. As a minimum, this will include claims paid during the year, for each origin year. Let $C_{ij}$ be claims paid for origin year $i$ and development year $j$ and let $n$ be the ultimo year, when all claims are finalized. We need to simulate a new diagonal in the development triangle; in Table 2.1, the simulated random variables are marked with bold-face. For long-tailed business, it might be that no origin years are finalized, in which case we add an observation $C_{i,n+1}$ to year 1.

There are several possibilities to generate a new diagonal and this is really outside the scope of this paper; one possibility is bootstrapping, see for e.g. Björkwall et al. (in press) where both non-parametric and parametric bootstrap is discussed. Another possibility is to simulate from a normal or log-normal distribution with mean given by the best estimate and variance given by formula (3) of Mack (1993). Of course, any other distribution that fits the data may be used.

The sum along the new diagonal yields the claims paid during the year for historical origin years $\tilde{C}_1 = \sum_{i=2}^{n} C_{i,n-i+2}$.

For some reserving methods $A_1$ we will also need a new diagonal of claims incurred or other quantities, e.g. when using the Munich Chain Ladder of Quarg and Mack (2005) or the method by Schnieper (1991). We will not go into the details here, but just assume that all variables needed by the actuary for reserving can be simulated for the new year.

2.1.3. Step 3: Best estimate of the closing reserve

In this step we generate the closing reserve $\tilde{R}_1$, for historical years as estimated by the end of the risk year. The idea is, for each simulated outcome in Step 2, to calculate $\tilde{R}_1$ by the same method $A_1$ as in Step 1, which is (an approximation to) the reserving method used in practice when calculating $R_0$.

We now have all the components needed to calculate the claims development result (CDR) $\tilde{T}$ in Eq. (2.1), for each of our, say, $B$ simulations. The empirical distribution of these CDRs is our estimate of the probability distribution of $\tilde{T}$. The $B$ run-off results can also be used further on in a risk model to interact with other risks in order to get the total risk and other aggregated risks. From the probability distribution of $\tilde{T}$ we can, of course, get the standard deviation, any Value-at-Risk (VaR) figure, or whatever risk measure we choose.

Note. In many cases one would wish to include (claims) inflation in the above calculations by initially adjusting the paid claims triangle for historic inflation and at the end of the calculations recalculate the result in running prices, by using some assumed future inflation. The outcome of the inflation for the risk year, as well as the assumed future inflation by the end of the year, should preferably be stochastic. We will not go into further details on inflation here.

2.2. Discussion

A problem with the one-year risk is that the reserves for long-tailed business might change so little over one year that "(...) it should not be a surprise that some long-tail business – where adverse movements in claims provisions emerge slowly over many years – require less solvency capital than some short-tail business (...) ". (AISAM-ACME, 2007). This phenomenon is also discussed and illustrated by a simple clarifying example in Dhaene et al. (2008), Section 7.

This phenomenon is a consequence of the one-year horizon of the Solvency II framework, which relates to risks that could appear in the financial statements over one year and does not take the long-term nature of insurance into account, right or wrong. Of course, mixing an ultimo perspective for liabilities with a one-year perspective for assets is not an alternative if we are interested in the combined total risk of the company.

3. The one-year premium risk

Let $\tilde{P}$ be the earned premium expected for the next calendar year. The premium risk relates to the event that $\tilde{P}$ will not be enough to cover the costs for the new risk year, i.e., it is the risk in the profit/loss statement at the end of the year. Let $E$ denote the operating expenses, and recall that $C_1$ is this years payments for the new risk year, and that $R_1$ is the (closing) claims reserve for the risk year. The profit/loss result is then

$$\tilde{T} = \tilde{P} - E - (\tilde{C}_1 + \tilde{R}_1).$$

(3.1)

If premium cycle variation was modelled, $\tilde{P}$ would be stochastic (the part coming from the premium reserve is always deterministic though). We will consider the premium as fixed here, i.e. we model the risk inherent in the premium volume the company expects to receive; we also assume that expenses will equal the budgeted value so that $E$ is fixed. The Value-at-Risk is then, $\text{VaR}(-\tilde{T}|D_0)$, where the randomness comes from the payments $\tilde{C}_1$ and the estimated reserve $\tilde{R}_1$. Here we have an expected profit $E(\tilde{T}|D_0)$ and as before VaR is the relative VaR, i.e. the 99.5% percentile in the distribution of $E(\tilde{T}|D_0) - \tilde{T}$, conditional on $D_0$.

In the QIS4 technical specifications, CEIOPS (2008), premium risk is introduced as follows.

TS.XIII.B.4 Premium risk relates to policies to be written (including renewals) during the period, and to unexpired risks on existing contracts.

In TS.XIII.B.12, the historic loss ratios, from which the volatility is calculated, are defined as the estimated cost at the end of the first development year, divided by earned premiums, in our notation $(C_{1,1} + R_{1,1})/P$. Basing the risk calculation on this loss ratio is consistent with our approach above.

There is a need to clarify the role of the premium reserve in this context. In today’s accounting, the premium reserve is computed pro rata temporis and then an additional provision for unexpired risks is added, if so required by the outcome of a liability adequacy test. While the definition above seems to imply that we must look at more than one years worth of premiums, this is not the case under strict pro rata temporis, since the opening and closing reserve will more or less cancel. However, the possible need for an unexpired risk add-on in the closing reserve should increase the premium risk. Hence, both our definition above and the QIS4 are implicitly based on the approximation that the probability of needing an unexpired risk add-on at time 1 is negligible.

There is an ongoing discussion on how to calculate technical provisions in the future, see IASB (2007) and the extensive discussion following the release of that discussion paper. From an
economic perspective, the premium reserve is not very different from the claims reserve, only that it relates to claims that have not yet occurred, but for which the company has a contractual liability. By this view, the premium reserve should be set up to cover expected costs for the part of next years risks that it covers, in our experience 10%–45% of a full year depending on the LoB. Note that this approach is like letting the liability adequacy test determine the premium reserve completely, abandoning the pro rata temporis perspective. In particular, the economic approach means that the company can recognize some of the profit at inception. This is quite controversial and when this is written, IASB has not yet decided on the calculation of (what is today called) the premium reserve.

With an economic perspective on the premium reserve, one-year risk models should ideally include the premium reserve and we now discuss how this can be done. Let \( U_t; t = 0, 1 \) be the opening and closing premium reserve, or more properly the CBNI (covered but not incurred) reserve. Let \( P \) be the expected premium income for the next year. Then the profit/loss result for the current year is

\[
\bar{T} = U_0 + P - E - (\bar{C}_1 + \bar{R}_1) - U_1.
\]  (3.2)

To find the probability distribution of \( \bar{T} \), and in particular \( \text{VaR}(\bar{T})(\varnothing) \), we will again refrain to simulation. There are several possibilities to simulate the claim cost \( (\bar{C}_1 + \bar{R}_1) \), one possibility being to simulate the corresponding loss ratio, with volatility \( \text{VaR} \). To find the probability distribution of \( \text{income} \) \( P \) opening and closing premium reserve, or more properly the CBNI (covered but not incurred) reserve, let \( P \) be the expected premium income for the next year. Then the profit/loss result for the current year is

\[
\bar{T} = U_0 + P - E - (\bar{C}_1 + \bar{R}_1) - U_1.
\]  (3.2)

To find the probability distribution of \( \bar{T} \), and in particular \( \text{VaR}(\bar{T})(\varnothing) \), we will again refrain to simulation. There are several possibilities to simulate the claim cost \( (\bar{C}_1 + \bar{R}_1) \), one possibility being to simulate the corresponding loss ratio, with volatility estimated as in QIS4. Kaufmann et al. (2001) suggest simulating the ultimo loss ratio (from historical loss ratios) and thus get the total claims cost, then split that cost into paid the first year and claims reserve, by using a beta distribution for the proportion paid the first year, see their equation (2.27). Yet another possibility is to simulate frequency and severity separately.

Since we condition on \( \varnothing \), the opening reserve \( U_0 \) is non-random, calculated by some method \( A_2 \). Similar to the case with the claims reserve, we suggest that for each of the 8 simulations, \( U_1 \) is computed by the same rule \( A_2 \), but now on the data including the simulated year. (\( A_2 \) would be similar to a liability adequacy test.)

Until the new accounting rules are clear, one might choose to stick to the simplified profit/loss result in (3.1).

4. Risk margins and discounting

In the Solvency II framework as well as in the forthcoming accounting standards IFRS 4 phase II as indicated in IASB (2007), best estimates of reserves includes discounting by a yield curve of risk-free interest rates. Furthermore, a risk margin is added to the reserves. In this section, we shall investigate how discounting and the adding of a risk margin influence the one-year insurance risk. The reader should note that this section is tentative and not based on our own risk modelling practice.

In QIS4, the risk margin is computed per segment and diversification effects between segments are not taken into account when aggregating; we will use the same approach. Within segments (LoB), we will assume that only one risk margin is computed, i.e. diversification between the premium and reserve risk is taken into account. If it is preferred to calculate a global risk margin for the total technical provisions of the company, the risk margin in this paper could be thought of as being derived by some special method for allocating the diversification effect to the LoB, such as the Shapley method, see e.g. Land et al. (2001).

We first add (2.1) and (3.2) to get the profit/loss result due to both historical years and the new year, putting an extra subindex \( d \) to all discounted quantities, i.e. to all reserves, including the risk margin which is part of the reserve and hence discounted, too. We then add the combined risk margin \( M^d \) for the premium reserve and claims reserve; note that this quantity is also discounted. Due to discounting, the expected CDR will tend to be negative; this calls for adding investment income transferred from financial operations, called \( I_t \) here; note that this amount is stochastic if computed as suggested in Section 4.1. Recall that \( C_1 = \bar{C}_1 + \bar{C}_1 \) is claims paid during year \( t = 1 \) for both historical years and the new year. The profit/loss from the segment, including the CDR, is now, recalling that the outgoing claims reserve is \( R_t = \bar{R}_1 + \bar{R}_1 \).

\[
T = (U_0^d + R_0^d + M_0^d) + P + I_1 - E - C_1 - (U_1^d + R_1^d + M_1^d). \]  (4.1)

Conceptually the risk margin is the extra amount, besides the expected value, a third party would demand for a transfer of the liabilities. (The risk margin is sometimes called the market value margin.) If reserves were traded on a liquid market, the probability distribution of any risk margin could be estimated from observations on that market. In reality such markets do not exist and some proxy must be used. In the Solvency II development, the preferred method is Cost-of-Capital (CoC) see EU Commission (2007); CoC is also one of the possibilities listed by IASB (2007) for IFRS 4, phase II.

In the CoC method, \( M^d \) is the cost of the solvency capital requirement (SCR) for running off the liabilities completely, i.e. the cost of holding the sum of the consecutive capital amounts required to run the business for each run-off year until the ultimate year. Since that capital for each year is the risk in the one-year result, it may look as though there is a circular reference here: the SCR could depend on the risk margin while the risk margin depends on the SCR. However, this is not the case, due to shifted time perspectives as will be shown in Section 4.2, where we go into the details for the CoC approach.

4.1. Investment income transferred from financial operations

The investment income transferred from financial operations \( I_t \) should be determined to meet the discounting, so that if all other things were equal (in particular the cash-flows equal their expected values), the claims development result (CDR) would be zero. After revaluation of the reserve at the end of the year, with the new interest rate but with the same expected cash-flows as before, the quantity \( I_t \) transfers the necessary amount to get a zero CDR from the financial result to the technical result. Financial income above (or below) this benchmark constitutes the financial result.

Technically this can be done with a replicating portfolio, i.e. a portfolio that meets the expected cash-flows of the liabilities with zero-coupon bonds of corresponding maturities. This portfolio is not locked-in but rather revaluated under the interest rate at the end of the year. \( I_t \) is the difference in value of this portfolio at time 0 and time 1, assuming that this years payment and the expected future cash-flows are the same as expected at the beginning of the year. This means that \( I_t \) will include not only one year of interest on the entire portfolio, but also the result from the change in interest rate from time 0 to 1. For further discussion on these matters, see Swiss Re (2001).

From a risk perspective, this means that \( I_t \) is stochastic given \( \varnothing \), due to the stochastic nature of the interest rate at time 1, but it is correlated to \( \bar{R}_1 \) in such a way as to transfer most of the interest risk to the asset side. The replicating portfolio represents the expectation of \( \bar{C}_1 \) and \( \bar{R}_1 \) at time 0; the revaluation of \( \bar{R}_1 \) contributes to the reserving risk and this risk will be altered somewhat by discounting. This is the remaining interest risk included in the insurance risk; it could be expected to be of less importance since it only operates on \( \bar{R}_1 - E(\bar{R}_1) \).
4.2. The Cost-of-Capital method

In principle, the CoC method requires a risk calculation for all years until complete run-off, so the time scale will now be \( t = 0, 1, 2, \ldots, n \), where \( n \) is the ultimate year of the run-off of the company’s liabilities at time \( t = 0 \).

To compute the risk margin, we consider the portfolio to be in run-off at the beginning of year 1, i.e., no premiums are written. Recall that \( R^d_n \) denotes the discounted claims reserve at the closing of year \( t \), and \( M^d_t \) is the corresponding risk marginal.

The run-off profit/loss \( T_t \) for year \( t \) is then

\[
T_t = R^d_{t-1} + M^d_{t-1} + I_t - C_t - R^d_t - M^d_t.
\]  

(4.2)

For simplicity in notation, at \( t = 0 \) we now let \( R^d_0 \) include the premium reserve as well; for \( t \geq 1 \) there is obviously no premium reserve. Note that the payments \( C_t \) during the year are quite rich to the contractual liabilities only, since there are no new policies written in the run-off situation.

Let SCR\(^{-1} \) denote the solvency capital set up at the closing of year \( t = 1 \), required to run the business during year \( t \), so that

\[
\text{SCR}\(^{-1} \) = \text{VaR}(\text{VaR}(I_t | D_{t-1})).
\]

(4.3)

where we have used the fact that \( R^d_m \) and \( M^d_{m-1} \) are non-random when we condition on \( D_{m-1} \), and hence only contribute to the expectation. Note that, since the best estimate is unbiased, and \( I_t \) is the capital income that makes the discounting, we have

\[ E(T_t | D_{t-1}) = M^d_t, \]

which is exactly the cost of providing the capital SCR\(^{-1} \), so that after this amount has been withdrawn, the expectation is zero.

For \( t = 1, 2, \ldots, n - 1 \), first note that the risk margin in the accounts at \( t = 1 \) is the one we expect to need at the closing of year \( t \), plus the CoC for year \( t \), i.e.,

\[
M^d_{t-1} = E(M^d_t | D_{t-1}) + \alpha \text{VaR}(\text{VaR}(I_t | D_{t-1})),
\]

(4.4)

where \( \alpha \) is the CoC rate above risk-free interest rate, often chosen to be 6%. These quantities are all known (non-random) at time \( t - 1 \) and so

\[
\text{VaR}(\text{VaR}(I_t | D_{t-1})) = \text{VaR}(C_t + R^d_t + M^d_t - I_t | D_{t-1}).
\]

(4.5)

Together with (4.4), we get an equation system that is, in principle, solvable by backwards recursion, where the starting value is the SCR\(^{-1} \) that was found in (4.3).

This shows that, as claimed earlier, there is no circular reference in letting the risk margin enter into the SCR calculation. However, solving the above equations is impracticable, even with simulation: let us look at the next-to-last year and try to find SCR\(^{-2} \) that was found in (4.3).

\[
\text{SCR}\(^{-2} \) = \text{VaR}(C_{n-1} + R^d_{n-1} + \alpha \text{VaR}(\text{VaR}(I_t | D_{t-1}) - I_t | D_{t-2})].
\]

If the computation of \( \text{VaR}(\text{VaR}(I_t | D_{t-1}) \) requires simulation, then for each outcome of the random variables in year \( n - 1 \), we must do a complete simulation of year \( n \). So if we use \( B \) simulations in general, we need \( B \times B \) iterations. Going one more year backwards requires \( B \times B \times B \) simulations, etc. This calls for simplification.

4.2.1. Simplified CoC method, using duration

A common simplification of the CoC calculation is to assume that the SCR in each year is run off at the same expected rate as the reserve. This approach is e.g. recommended in Section 3 of CEA (2008). Its origin, as well as the origin of the CoC itself, seems to be with the Swiss solvency test, see Keller (2006). To the best of our knowledge, no study has been published on the potential difference between the simplified approach and the original one; the simplified approach is rather motivated by the fact that the original approach is “unnecessarily complex and in practice very hard for companies to perform” (CEA, 2008, Section 3).

Assume that there is a known payment pattern \( p_1, p_2, \ldots, p_n \), with \( \sum_{t=1}^{n} p_t = 1 \); in practice this would be estimated from the claims triangle in \( D_0 \) as part of the process of computing the best estimate. Then \( \sum_{t=1}^{n} p_t \) is the part of the best estimate that remains at the beginning of year \( t \), and the approximation is that \( \text{SCR}\(^{-1} \approx \sum_{t=1}^{n} p_t \times \text{SCR}\(^0 \)). The CoC rate \( \alpha \) is assumed fixed (say at 6% above risk-free rate). By adding all future costs of capital we arrive at the risk margin before discounting as \( M_0 = \alpha \times c \times \text{SCR}\(^0 \), where the calculation of SCR\(^0 \) will be discussed below and \( c \) is given by

\[
c = \sum_{t=1}^{n} p_t = \sum_{s=1}^{n} \sum_{t=1}^{s} p_t = \sum_{s=1}^{n} s p_s.
\]

(4.6)

If the SCR is continuously lowered during the year, rather than kept for the entire year, then 0.5 should be drawn from \( c \) which is then simply the (average) duration of the reserve. The factor \( c \) equals the duration plus 0.5 if we assume payments to be made at July 1, but initial SCR recalculated only at the beginning of the year. In any case, an interesting way of looking at the well-known approximation of the SCR needed for CoC is to say that we are holding the entire initial SCR\(^0 \) for as long as the (average) duration of the liabilities (plus 0.5 years when relevant).

The simplified approach implies that \( M^d_t \) is non-stochastic; we conclude that we may leave the risk margin out of the SCR calculation, as is done in QIS4. The initial SCR\(^0 \) that we need for the CoC calculation would then result from

\[
\text{SCR}\(^0 \) = \text{VaR}(C_t + R^d_t - I_t | D_0).
\]

(4.7)

Here it should be remembered that we use the relative VaR, so that SCR\(^0 \) is really computed as the 99.5% quantile of \( L = C_1 + R^d_1 - I_1 - R^d_0 \), which has \( \text{VaR}(L | D_0) = 0 \). The simulation approach to this kind of risk calculation was discussed in the first two sections of this paper. This finishes the specification of the simplified CoC approach.

Note. While the risk margin may thus be left out of the SCR-calculation, it must be taken into account when calculating the own funds in the balance sheet, since it is a fundamental part of the approximation of a market value of the reserves.

5. Conclusions

In this paper, we have tried to clarify the one-year perspective on reserve risk. We have demonstrated that for the Cost-of-Capital (CoC) method, there is no circular reference in letting the risk margin enter into the solvency capital requirement (SCR) calculations. In practice, one has to use a simplified method, for which we have seen that we can omit the risk margin completely from the SCR calculations.

We have also given the simplified CoC method an interpretation in terms of reserve duration, which might turn out to be useful.

As concerns the premium reserve, we have initiated a discussion of its role in the one-year premium risk and indicated very briefly how a simulation approach might include the premium reserve risk. In this area there is need for further research and numerical tests, before the ideas can be implemented.
Acknowledgements

This work has benefitted a lot from discussions with Peter England, EMB, who introduced the basic ideas of the one-year reserve risk. The authors are also grateful to Jörgen Olsén, Guy Carpenter, and Susanna Björkwall, Länsförsäkringar Alliance, for valuable comments on a preliminary version of this paper.

References